# DILATONIC BLACK HOLES IN HETEROTIC STRING THEORY

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We obtain the dilaton solution in the background of a spherically symmetric black hole with curvature-squared corrections in arbitrary $d$ spacetime dimensions and a spherically symmetric black hole solution with dilatonic charge and curvature-squared corrections in heterotic string theory compactified on a torus. For this black hole we obtain its entropy, temperature, specific heat and mass.

The most general static, spherically symmetric metric in $d$ spacetime dimensions can be written as ($f, g$ being arbitrary functions of the radius $r$)

$$d\,s^2 = -f(r)\,d\,t^2 + g^{-1}(r)\,d\,r^2 + r^2\,d\,\Omega^2_{d-2}. \tag{1}$$

For pure Einstein-Hilbert gravity in vacuum, the solution to the Einstein equations gives ($r_H$ being the horizon radius)

$$f(r) = g(r) = 1 - \left(\frac{r_H}{r}\right)^{d-3}. \tag{2}$$

We are interested in extending this solution in the presence of a dilaton, considering string-theoretical $\alpha'-$corrections to first order in $\alpha'$. We are focusing in particular in $\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}$ corrections. The effective action we are considering is

$$\frac{1}{16\pi G}\int\sqrt{-g}\left(\mathcal{R} - \frac{4}{d-2}\left(\partial^\mu\phi\right)\partial_\mu\phi + \mathrm{e}^{\frac{4}{d-2}\phi}\frac{\lambda}{2}\,\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}\right)\mathrm{d}^dx. \tag{3}$$

Here $\lambda = \frac{\alpha'}{2}, \frac{\alpha'}{4}$ and 0, for bosonic, heterotic and type II strings, respectively. We are only considering gravitational terms: we can consistently settle all fermions and gauge fields to zero for the moment. That is not the case of the dilaton, as it can be seen from the field equations (neglecting terms which are quadratic in $\phi$):

$$\nabla^2\phi - \frac{\lambda}{4}\,\mathrm{e}^{\frac{4}{2-d}\phi}\left(\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right) = 0, \tag{4}$$

$$\mathcal{R}_{\mu\nu} + \lambda\,\mathrm{e}^{\frac{4}{2-d}\phi}\left(\mathcal{R}_{\mu\rho\sigma\tau}\mathcal{R}_\nu{}^{\rho\sigma\tau} - \frac{1}{2(d-2)}g_{\mu\nu}\mathcal{R}_{\rho\sigma\lambda\tau}\mathcal{R}^{\rho\sigma\lambda\tau}\right) = 0. \tag{5}$$

For the particular, spherically symmetric case we are considering, we take (1) with $f(r) = g(r)$ given by (2) as the $\lambda = 0$ metric. We are interested in computing the first $\alpha'$ corrections to $\phi$ and $g_{\mu\nu}$, using (4) and (5) and always working perturbatively in $\lambda$, neglecting $\lambda^2$ and higher order terms. We can take the $\lambda = 0$ metric in (4) in order to compute both $\nabla^\mu \nabla_\mu \phi$, since $\phi$ is of order $\lambda$, and $\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}$, since this term is already multiplied by $\lambda$. Putting everything together, and integrating (4) twice, we get for its solution

$$\phi(r) = \frac{\lambda}{r_H^2}\frac{(d-2)^2}{4}\left[\ln\left(1-\left(\frac{r_H}{r}\right)^{d-3}\right) - \frac{d-3}{2}\left(\frac{r_H}{r}\right)^2 - \frac{d-3}{d-1}\left(\frac{r_H}{r}\right)^{d-1} + B\left(\left(\frac{r_H}{r}\right)^{d-3}; \frac{2}{d-3}, 0\right)\right]. \tag{6}$$

$B(x;a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\,dt$ is the incomplete Euler beta function.

Eq. (6) is the only solution for the dilaton in the background of a spherically symmetric black hole with $\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}$ corrections in $d$ dimensions which is regular at $r = r_H$. This dilaton solution acts as primary hair, since it does not introduce any new physical parameter besides the ones of the black hole. When one compactifies string theory to $d$ dimensions on a $(d_s - d)$–dimensional torus ($d_s = 26$ for bosonic and $d_s = 10$ for heterotic strings), the solution to (5) is of the form (1), with

$$g(r) = \left(1-\left(\frac{r_H}{r}\right)^{d-3}\right)\left[1-\lambda\frac{(d-3)(d-4)}{2}\frac{r_H^{d-5}}{r^{d-1}}\frac{r^{d-1}-r_H^{d-1}}{r^{d-3}-r_H^{d-3}}\right],$$
$$f(r) = g(r) + 4\left(1-\left(\frac{r_H}{r}\right)^{d-3}\right)\frac{d_s-d}{(d_s-2)^2}(\phi - r\phi'). \tag{7}$$

After integrating (3) in euclidian space, one obtains the black hole free energy

$$F = \left(1-\frac{d(d-3)}{2}\frac{\lambda}{r_H^2}\right)\frac{\Omega_{d-2}}{16\pi G}r_H^{d-3} \tag{8}$$

and entropy, by Wald's formula $S = -2\pi\int_H \frac{\partial\mathcal{L}}{\partial R^{\mu\nu\rho\sigma}}\varepsilon^{\mu\nu}\varepsilon^{\rho\sigma}\sqrt{h}\,d\,\Omega_{d-2}$, since we are dealing with a lagrangian $\mathcal{L}$ with higher derivatives. We get

$$S = \frac{1}{4G}\int_H\left(1+\frac{\lambda}{r_H^2}(d-3)(d-2)\right)\sqrt{h}\,d\,\Omega_{d-2} = \frac{A_H}{4G}\left(1+(d-3)(d-2)\frac{\lambda}{r_H^2}\right).$$

The black hole temperature is given by $T = \lim_{r\to r_H}\frac{\sqrt{g}}{2\pi}\frac{d\sqrt{f}}{d\,r}$. In our particular case, this gives us $T = \frac{d-3}{4r_H\pi}\left[1+\frac{\lambda}{r_H^2}\delta T(d,d_s)\right]$, with[a]

$$\begin{aligned}\delta T(d,d_s) = \Big[&3d^5 + (-3d_s + 2\gamma - 18)d^4 + \left(-2d_s^2 - 2\gamma d_s + 26d_s - 10\gamma + 27\right)d^3 \\ &+ \left(12d_s^2 + 10\gamma d_s - 83d_s + 16\gamma + 28\right)d^2 \\ &- 2\left(9d_s^2 + 8\gamma d_s - 46d_s + 4\gamma + 38\right)d + 4\left(2d_s^2 + 2\gamma d_s - 7d_s + 8\right) \\ &+ 2(d-2)^2(d-1)(d-d_s)\psi^{(0)}\left(\frac{2}{d-3}\right)\Big]\frac{1}{4(d-1)(d_s-2)^2}.\end{aligned}$$

[a] The digamma function is given by $\psi(z) = \Gamma'(z)/\Gamma(z)$. For positive $n$, one defines $\psi^{(n)}(z) = \frac{d^n\,\psi(z)}{d\,z^n}$. This definition can be extended for other values of $n$ by fractional calculus analytic continuation. $\gamma$ is Euler's constant, defined by $\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^n \frac{1}{k} - \ln n\right)$, with numerical value $\gamma \approx 0.577216$.

$\delta T(d, d_s)$ is always negative for every relevant values of $d$ and $d_s$, which means $\alpha'$–corrections decrease $T$. This suggests that $T$ may reach a maximum, depending on $d$ and $d_s$. For every relevant values of $d$ and $d_s$ we computed that maximal temperature, which is always lower than the critical Hagedorn string temperature.

The specific heat is given by $C = T\frac{\partial S}{\partial T} = -(d-2)\frac{A_H}{4}\left[1 + \frac{\lambda}{r_H^2}\delta C(d, d_s)\right]$, with

$$\begin{aligned}\delta C(d, d_s) = &-\frac{d-2}{2(d-1)(d_s-2)^2}\left[3d^4 + (2\gamma - 3(d_s+4))d^3\right.\\ &- (2d_s(2d_s+\gamma-14) + 6\gamma+5)d^2 + (d_s(20d_s+6\gamma-91) + 4\gamma+82)d\\ &\left.+\ 62d_s - 4d_s(4d_s+\gamma) - 64 + 2(d-1)(d-2)(d-d_s)\psi^{(0)}\left(\frac{2}{d-3}\right)\right].\end{aligned}$$

$\delta C(d, d_s)$ is always positive for every relevant values of $d$ and $d_s$, which means $\alpha'$–corrected black holes keep being thermodynamically unstable.

The mass is given by $M = ST + F = \left[1 + \frac{\lambda}{r_H^2}\delta M(d, d_s)\right]\frac{(d-2)\Omega_{d-2}}{16\pi G}r_H^{d-3}$, with

$$\begin{aligned}\delta M(d, d_s) = &\left[3d^4 + (2\gamma - 3(d_s+4))d^3 + (2d_s(d_s-\gamma+2) - 6\gamma+19)d^2\right.\\ &+ (d_s(-10d_s+6\gamma+29) + 4\gamma-38)d + 2d_s(4d_s-2\gamma-17)\\ &\left.+\ 2((d-3)d+2)(d-d_s)\psi^{(0)}\left(\frac{2}{d-3}\right) + 32\right]\frac{(d-3)}{4(d-1)(d_s-2)^2}.\end{aligned}$$

The sign of $\delta M(d, d_s)$ depends on its parameters. For $d = 4$ and $d = 5, d_s = 10$ it is negative; for $d = 5, d_s = 26$ and $d > 5$ it is positive. In terms of $M$ and $T$, $F$ and $S$ are:

$$\begin{aligned}F = &\frac{M}{d-2}\left[1 - \frac{4^{\frac{d}{3-d}}(d-3)(d-2)\pi}{(d-1)(d_s-2)^2}\left(\frac{d-2}{M\Gamma\left(\frac{d-1}{2}\right)}\right)^{\frac{2}{d-3}}\lambda\left[3d^3 + (-3d_s+2\gamma-6)d^2\right.\right.\\ &+ \left(4d_s^2 - 2(5+\gamma)d_s - 2\gamma+15\right)d + d_s(-4d_s+2\gamma+17)\\ &\left.\left.+\ 2(d-1)(d-d_s)\psi^{(0)}\left(\frac{2}{d-3}\right) - 16\right]\right];\\ S = &\frac{2^{3-2d}\pi^{\frac{3-d}{2}}(d-3)^{d-2}T^{2-d}}{G\Gamma\left(\frac{d-1}{2}\right)}\left[1 + \frac{4(d-2)^2\pi^2\lambda T^2}{(d-3)^2(d-1)(d_s-2)^2}\left[(2\gamma-3(d_s+4))d^3\right.\right.\\ &+\ 3d^4 + (-2d_s(d_s+\gamma-10) - 6\gamma+3)d^2 + (d_s(12d_s+6\gamma-59) + 4\gamma+50)d\\ &\left.\left.-\ 2(d_s(5d_s+2\gamma-19) + 20) + 2((d-3)d+2)(d-d_s)\psi^{(0)}\left(\frac{2}{d-3}\right)\right]\right].\end{aligned}$$

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## References

1. F. Moura, arXiv:0912.3051 [hep-th].